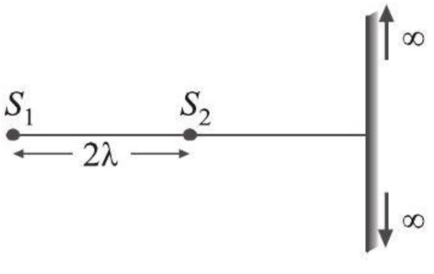
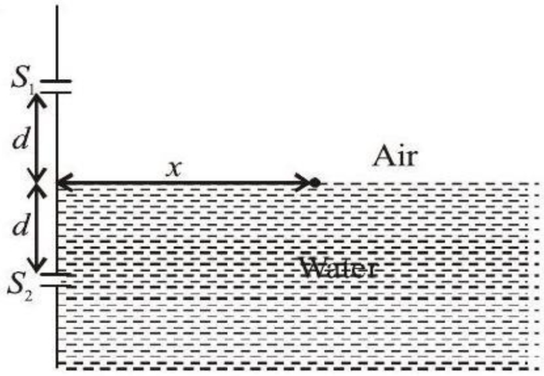
Wave Optics

1. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \leq \theta \leq 30^\circ$ is
2. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$ th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is
3. In a double-slit experiment, green light (5303Å) falls on a double slit having a separation of 19.44μ m and a width of 4.05μ m. The number of bright fringes between the first and the second diffraction minima is:
4. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be :
5. Calculate the limit of resolution (in radian) of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.
6. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance (in m) between the first dark fringes one other side of the central bright fringe is
7. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000Å is used, the minimum separation (in μm) between two points, to be seen as distinct, will be :
8. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_o is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_o/I) equals (nearly) :
9. There are two sources kept at distances 2λ. A large screen is perpendicular to line joining the sources. Number of maximas on the screen in this case is ($\lambda =$ wavelength of light)



10. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index $= \frac{4}{3}$) as shown in the figure. The positions of maximum on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index $= 1$), $2d$ is the separation between the slits and m is an integer. The value of p is



11. Two waves of the same frequency have amplitudes 2 and 4. They interfere at a point where their phase difference is 60°. Find their resultant amplitude.

12. In an interference pattern, at a point there observe 16^{th} order maximum for $\lambda_1 = 6000Å$. What order will be visible here if the source is replaced by light of wavelength $\lambda_2 = 4800Å$?

13. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm, coming from a distant object, the limit of resolution of the telescope (in radian) is

14. A young's double-slit arrangement produces interference fringes for sodium light ($\lambda = 5890Å$) that are 0.20° apart. What is the angular fringe separation (in degree) if the entire arrangement is immersed in water? (refractive index of water is $4/3$).

15. A beam of plane polarised light falls normally on a polariser (cross-sectional area 3×10^{-4} m^2) which rotates about the axis of the ray with an angular velocity of 31.4 rad/s. Find the energy of light (in joule) passing through the polariser per revolution if flux of energy of the incident ray is 10^{-3} W.

SOLUTIONS

1. **(641)** For 'n' number of maximas

$d \sin\theta = n\lambda$

$0.32 \times 10^{-3} \sin 30° = n \times 500 \times 10^{-9}$

$$\therefore\ n = \frac{0.32 \times 10^{-3}}{500 \times 10^{-9}} \times \frac{1}{2} = 320$$

Hence total no. of maximas observed in angular range

$-30° \le \theta \le 30°$

$= 320 + 1 + 320 = 641$

2. **(0.85)** Given, path difference, $\Delta x = \frac{\lambda}{8}$

Phase difference $(\Delta\phi)$ is given by

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\Delta\phi = \frac{(2\pi)}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

For two sources in different phases,

$$I = I_0 \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

$$= \frac{1 + \cos\frac{\pi}{4}}{2} = \frac{1 + \frac{1}{\sqrt{2}}}{2} = 0.85$$

3. **(5)**

4. **(4)** Given amplitude ratio of waves is $\frac{a_1}{a_2} = \frac{3}{1}$

$$\text{so, } \frac{I_{max}}{I_{min}} - \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2$$

$$= \left(\frac{\frac{a_2}{a_1} + 1}{\frac{a_2}{a_1} - 1}\right)^2 = \left(\frac{3+1}{3-1}\right)^2 = \left(\frac{4}{2}\right)^2 = \frac{4}{1} = 4$$

5. **(305×10^{-9})** $\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 500 \times 10^{-9}}{2}$

$= 305 \times 10^{-9}$ rad.

6. **(1.2×10^{-3})** $\sin\theta = \frac{\lambda}{d}$

$$\text{or}\quad \theta \approx \frac{600 \times 10^{-9}}{1 \times 10^{-3}}$$

$$\text{or}\quad \theta = 6 \times 10^{-4} \text{ rad}$$

$$\therefore\quad \beta = D\theta$$

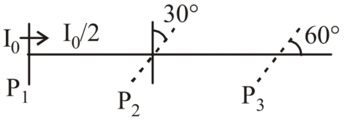
$$= 2 \times 6 \times 10^{-4} = 1.2 \times 10^{-3} \text{ m}$$

7. **(0.24)** $x = \dfrac{1.22\lambda}{2\mu\sin\theta}$

$= 0.24\ \mu m$

8. **(10.67)** $I = \left(\dfrac{I_0}{2}\right)\cos^2 30^\circ \cos^2 60^\circ$

$= \dfrac{I_0}{2} \times \dfrac{3}{4} \times \dfrac{1}{4}$



$\therefore\ \dfrac{I_0}{I} = \dfrac{32}{3} = 10.67$

9. **(3)** $\Delta x_{max} = 0$ and $\Delta x_{max} = 2\lambda$

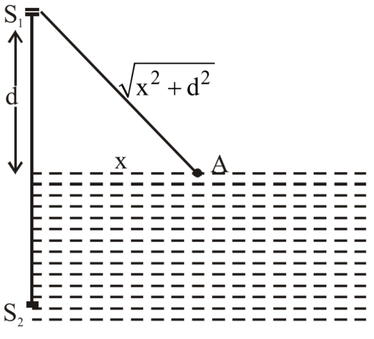
Theortical maximas are $= 2n + 1 = 2 \times 2 + 1 = 5$

But on the screen there will be three maximas.

10. **(3)** For maxima

Path defference $= m\lambda$

$\therefore\ S_2A - S_1A = m\lambda$



$\therefore\ \left[(n-1)\sqrt{d^2+x^2} + \sqrt{d^2+x^2}\right] - \sqrt{d^2-x^2} = m\lambda$

$\therefore\ (n-1)\sqrt{(d^2+x^2)} = m\lambda$

$\therefore\ \left(\dfrac{4}{3}-1\right)\sqrt{d^2+x^2} = m\lambda$

$\therefore\ \sqrt{d^2+x^2} = 3m\lambda$

$\therefore\ d^2 + x^2 = 9m^2\lambda^2$

$\therefore\ x^2 = 9m^2\lambda^2 - d^2$

$\therefore\ p^2 = 9 \Rightarrow p = 3$

11. **($\sqrt{28}$)** The resultant amplitude is given by

$$R = (a_1^2 + a_2^2 + 2a_1a_2\cos\phi)^{1/2}$$

$$= (2^2 + 4^2 + 2 \times 2 \times 4\cos 60^\circ)^{1/2}$$

$$= \sqrt{28}\,.$$

12. **(20)** The distance of n^{th} maxima from central maxima is given by

$$y_n = n\frac{D\lambda}{d},$$

For y_n to be constant, $n\lambda$ = constant. Thus

$$n_1\lambda_1 = n_2\lambda_2$$

$$\therefore \quad n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{16\times 6000}{4800} = 20$$

13. **(3×10^{-7})** $\theta = \frac{1.22\lambda}{d} = \frac{1.22\times 600\times 10^{-9}}{250\times 10^{-2}}$

$$= 3.0 \times 10^{-7} \text{ rad}$$

14. **(0.15)** If β be the fringe width in air, then in water

$$\beta_{water} = \frac{\beta}{\mu_w} = \frac{0.20°}{4/3}$$

15. **(10^{-4})** If I_0 is the intensity of plane polarised light incident on the polariser, then intensity of emerging light is given by

$$I = I_0\cos^2\theta$$

The average value of I over one revolution can be calculated as :

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} I d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_0\cos^2\theta d\theta$$

$$= \frac{I_0}{2}.$$

Intensity is given by

$$I_0 = \frac{\text{Power}}{\text{area}}$$

$$= \frac{10^{-3}}{3\times 10^{-4}} = \frac{10}{3}\text{ W/m}^2.$$

$$\therefore \quad I_{av} = \frac{I_0}{2} = \frac{5}{3}\text{W/m}^2.$$

The energy of light passing through the polariser per revolution

$$E = I_{av}\times A\times T = I_{av}\times A\times\frac{2\pi}{\omega}$$

$$= \frac{5}{3}\times(3\times 10^{-4})\times\frac{2\pi}{31.4}$$

$$= 10^{-4}\text{ J.}$$